NAASPH-12.2017-2018.2A Mid-term exam March 5 2018

Astroparticle Physics
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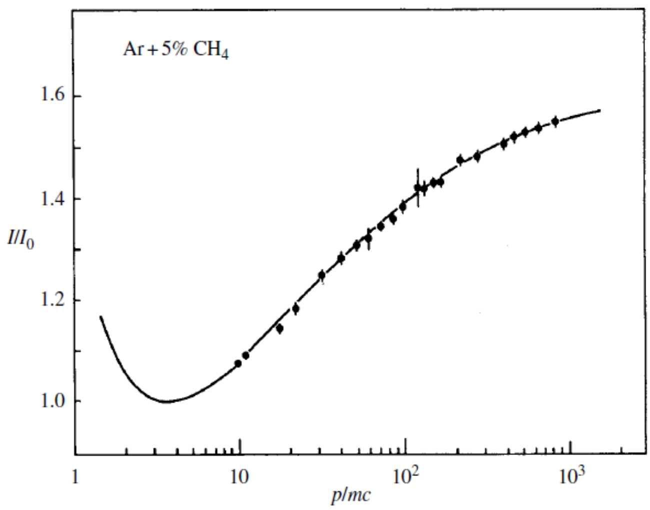
You don't have to use separate sheets for every question.
Write your name and S number on every sheet
There are **4 questions** with a total number of marks: 28

WRITE CLEARLY

(1) (Total 10 marks)
The mean ionization loss of a charged particle with mass m and momentum p in a medium with mass number A and atomic number Z is plotted in the figure below. Here I_0 is the minimum ionization energy loss, which is about 1,5 MeV cm^2 g^{-1}.



(a) (2 marks)
Proof that $\dfrac{p}{mc} = \beta\gamma$, where γ is the Lorentz factor and $\beta = \frac{v}{c}$.

(b) (2 marks)
What is the dominant mechanism for the energy loss in the region where $1 < \dfrac{p}{mc} < 1000$?

(c) (2 marks)
A muon with a kinetic energy of 10 GeV traverses a scintillator with a thickness of 2 cm. Assume for the muon rest mass a value of 100 MeV c^{-2}.

Calculate the value of $\dfrac{p}{mc} = \beta\gamma$.

(d) (2 marks)
The density of the scintillator is 1 g cm^{-3}. Calculate the energy loss of this muon while passing through the medium.

(e) (2 marks)
Describe a mechanism which can be used to observe the passage of the muon through the plastic, taking advantage of the amount of energy lost in the medium.

(2) (Total 3 marks)
Muons are produced in the high atmosphere, as a result of the collisions of cosmic rays with atmospheric nuclei. Compute the minimum kinetic energy at which muons must have been produced to reach the surface of the Earth, assuming that they are produced 20 km above the sea level and are arriving from the zenith.
Consider the lifetime $\tau_\mu = 2.2\ \mu$s and the mass $m_\mu c^2 = 106$ MeV.

(3) (Total 9 marks)
In the frame of the cosmological principle, the universe expansion can be described by the Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{R^2}$$

Use the fluid equation $\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + P) = 0$ and the equation of state $P = w\rho$, to answer the following questions:

(a) (2 marks)
Show that the energy density and the scale parameter R are related by $\rho \sim R^{-3(w+1)}$.

(b) (2 marks)
For a radiation-dominated universe, give the equation of state and the relation between the energy density ρ and the scale parameter R.

(c) (2 marks)
For a matter-dominated universe, write the equation of state and the relation between the energy density ρ and the scale parameter R.

(d) (3 marks)
Verify that, for $k = 0$, the age of the universe is given by: $t_0 = \frac{2}{3(w+1)H_0}$, where $H_0^2 = \frac{8\pi G\rho_0}{3}$.

(4) (Total 6 marks)

(a) (3 marks)
Find the expression for the dependence of the time t of the density ρ for an expanding flat universe ($k = 0$), dominated by radiation.

(b) (3 marks)
Show that the temperature scales with $t^{-1/2}$.

Solutions

Question 1.a. To answer the question **a** we have to consider that the momentum is given by $p = mv\gamma = m\beta c\gamma \rightarrow \frac{p}{mc} = \frac{m\beta c\gamma}{mc} = \beta\gamma$.

Question 1.b: the dominant mechanism is ionization.

Question 1.c: For a muon with energy 10 GeV we have:

$$\gamma = \frac{E}{m} \sim \frac{p+m}{m} = \frac{10^10eV + 10^8eV}{10^8eV} = 101$$

The relativistic speed is given by: $\beta = \sqrt{\frac{\gamma^2-1}{\gamma^2}} = 0.99995 \rightarrow \beta\gamma = 101$.

Question 1.d: The energy loss is given by $\Delta E = t\frac{dE}{dx}$, where $t = 2cm$ $\rho = 2\ g\ cm^{-2}$. To compute the $\frac{dE}{dx}$ we have to take the value of $\frac{I}{I_0}$ from the plot, that is $\frac{I}{I_0}(\beta\gamma = 101) = 1.42$.
Then $I = \frac{dE}{dx} = I_0 \cdot 1.42 = 2.13\ MeV/(gcm^{-2})$
Finally $\Delta E = t\ \frac{dE}{dx} = 2g\ cm^{-2} \cdot 2.13\ MeV/(g\ cm^{-2}) = 4.26\ MeV$.

Question 1.e: Scintillation light .

Question 2. In the laboratory frame, the muon produced at the top of the atmosphere travels over a distance $L = \gamma\beta c\tau$. We can write:

$$\beta\gamma = \frac{L}{c\tau} = \sqrt{\gamma^2 - 1} \rightarrow \gamma^2 = \frac{L}{c\tau}^2 + 1 \rightarrow \gamma \simeq \frac{L}{c\tau}$$

This last equation is true because $\frac{L}{c\tau} >> 1$.
Finally, we can use the expression of γ to compute the kinetic energy, that is given by (c=1):

$$K = (\gamma - 1)m = 3\ GeV$$

Question 3.a Combining the fluid equation $\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + P) = 0$ and the equation of state $P = w\rho$, we get:

$$\dot{\rho} = -3(w+1)\ \rho\ \frac{\dot{R}}{R} \rightarrow \frac{\dot{\rho}}{\rho} = -3(w+1)\frac{\dot{R}}{R}$$

so we get to the answer to question a:

$$\frac{\rho}{\rho_0} = (\frac{R}{R_0})^{-3(w+1)}$$

Question 3.b In the case of a matter-dominated universe we have $P = 0$, so that $w = 0$, leading to:$\rho \sim R^{-3}$

Question 3.c For a radiation-dominated universe $w = 1/3$, then $\rho \sim R^{-4}$.

Question 3.d To estimate the age of the universe in the case of a flat universe we have to go back to Friedmann equation, with $k = 0$.

$$H^2 = \frac{8\pi G}{3}\rho = \frac{\dot{R}^2}{R^2}$$

Organizing both sides we get:

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2$$

where we can substitute the answer to question **3.a**, and we get:

$$\dot{R}^2 = \left[\frac{dR}{dt}\right]^2 = \frac{8\pi G\rho_0}{3}\frac{R_0^{3(w+1)}}{R^{3(w+1)}}R^2$$

Taking the positive root we get:

$$dt = \sqrt{\frac{3}{8\pi G\rho_0 R_0^{3(w+1)}}R^{3w+1}}dR$$

re-arranging the two terms gives the following:

$$\sqrt{\frac{8\pi G\rho_0 R_0^{3(w+1)}}{3}}\ dt = R^{\frac{(3w+1)}{2}}\ dR$$

Integrating gets:

$$\sqrt{\frac{8\pi G\rho_0}{3}}R_0^{\frac{3}{2}(w+1)}\ t = \frac{R^{\frac{3}{2}(w+1)}}{\frac{3}{2}(w+1)}$$

Finally:

$$t = t_0\left(\frac{R}{R_0}\right)^{\frac{3}{2}(w+1)}$$

where t_0 is a constant term, defined as follows:

$$t_0 = \frac{1}{\frac{3}{2}(w+1)}\sqrt{\frac{3}{8\pi G\rho_0}}$$

Given that $H_0^2 = \frac{8\pi G\rho_0}{3}$, we have the required result:

$$t_0 = \frac{2}{3(w+1)H_0}$$

Question 4.a Neglecting the curvature term and using the relation between the energy density and the scale factor for radiation-dominated universe, namely $\rho_r \sim R^{-4}$, we get:

$$\frac{\dot{\rho}_r}{\rho_r} = -4(\frac{\dot{R}}{R}) = -4\sqrt{\frac{8\pi G\rho_r}{3}}$$

If denote the constant factor by $\alpha = -4(\frac{8\pi G}{3})$, we have:

$$\dot{\rho}_r = \frac{d\rho_r}{dt} = \ \alpha\ \rho_r^{3/2} \rightarrow \rho_r^{-3/2}\ d\rho_r = \ \alpha\ dt$$

Integrating the above equation we can find the relation between the density and the time:

$$\rho_r^{-1/2} \sim t \rightarrow \rho_r\ \sim \frac{1}{t^2}$$

Question 4.b By considering Stefan's law, $\rho_r c^2 = \frac{4\sigma T^4}{c}$, we can rewrite the equation and get:

$$\rho_r\ \sim \frac{1}{t^2} \rightarrow T \sim \frac{1}{\sqrt{t}}$$